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Spin-3/2 Fields in Flat Space-Time

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The equations for the spin-3/2 (Rarita–Schwinger) field given by linearized simple supergravity are written in space-plus-time form in terms of $SU(2)$ spinors, assuming that the background space-time is flat. Some consequences of these equations are analyzed and a Hamiltonian structure for the Rarita–Schwinger field is obtained.

1. INTRODUCTION

Supergravity theory gives a consistent system of equations for a spin-3/2 (Rarita–Schwinger) massless field coupled to a gravitational field. The spin-3/2 field is represented by anticommuting variables and generates the torsion of the connection. The supergravity field equations are invariant under supersymmetry transformations, which mix the spin-3/2 field with the variables representing the gravitational field (see, e.g., van Nieuwenhuizen, 1981).

When the simple supergravity field equations are linearized with respect to the spin-3/2 fields about a solution with vanishing spin-3/2 fields, one obtains a consistent equation for the Rarita–Schwinger field on a solution of the Einstein vacuum field equations. In this approximation, the supersymmetry transformations affect only the spin-3/2 field and the torsion of the connection vanishes (see, e.g., Aichelburg and Urbantke, 1981; Güven, 1980; Torres del Castillo, 1989a).

In this paper we consider the equations for the Rarita–Schwinger field on flat space-time, given by the linearized supergravity field equations, on a semiclassical level (in particular, we shall assume that the spinor components

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of the Rarita–Schwinger field are ordinary complex-valued functions). The equations for the spin-3/2 field are written in space-plus-time form and by expanding a gauge-invariant field made out of the Rarita–Schwinger field in plane waves, we obtain a Hamiltonian structure that does not require the theory of constrained Hamiltonian systems [in fact, as shown in Torres del Castillo and Acosta-Avalos (1994), by this procedure one can find an infinite number of different Hamiltonian structures]. In Senjanović (1977) and Pilati (1978) a Hamiltonian formulation for the Rarita–Schwinger field is given following the standard procedure starting from a (Lorentz-invariant) Lagrangian density for the spin-3/2 field, which leads to the appearance of constraints and therefore to the necessity of using Dirac brackets instead of Poisson brackets in order to take into account the existence of constraints. In the approach followed here, which is simpler and more elementary, the Hamiltonian structure and the Hamiltonian functional are deduced from the field equations themselves, avoiding the use of Lagrangians (and the presence of the concomitant constraints) and without having to choose a specific gauge. The Hamiltonian structure so obtained is then employed to analyze some conserved currents arising from the equations for the spin-3/2 field. In Section 2 we start from the equations for the spin-3/2 field written in terms of two-component spinors in four dimensions and we express them in space-plus-time form, in terms of spinors in three dimensions (cf. also Sommers, 1980; Sen, 1981). In Section 3, we obtain a Hamiltonian structure for the Rarita–Schwinger field corresponding to an indefinite Hamiltonian functional. In most of this paper we employ three-dimensional spinors following the notation and conventions of Torres del Castillo (1992, 1994a,b).

2. THE RARITA–SCHWINGER EQUATION IN 3-PLUS-1 FORM

If the field equations of simple supergravity are linearized with respect to the spin-3/2 fields about a solution with vanishing spin-3/2 fields, apart from the Einstein vacuum field equations with a torsion-free connection, the following equations for a spin-3/2 field are obtained (Aichelburg and Urbantke, 1981):

$$\nabla_{A\dot{B}}\psi^{A}{}_{C\dot{D}} = \nabla_{C\dot{D}}\psi^{A}{}_{A\dot{B}} \tag{1}$$

Equations (1) are free from algebraic constraints and are invariant under the supersymmetry transformations

$$\psi_{AB\dot{C}} \to \psi_{AB\dot{C}} + \nabla_{B\dot{C}}\alpha_{A} \tag{2}$$

where α_A is an *arbitrary* spinor field, provided that the Ricci tensor vanishes. In what follows, we shall assume that the background space-time is flat.

In order to decompose the field equations (1) in space-plus-time form, we shall introduce a future-pointing unit vector normal to the constant-time hypersurfaces, $n_{A\dot{B}}$; thus,

$$n^{A\dot{B}} n_{A\dot{B}} = 2 \tag{3}$$

The null tetrad vectors $\partial_{A\dot{B}}$ (which satisfy the condition $\partial_{A\dot{B}} \cdot \partial_{C\dot{D}} = -2\epsilon_{AC}\epsilon_{\dot{B}\dot{D}}$) can be expressed as

$$\partial_{A\dot{B}} = -\sqrt{2}\, n^B{}_{\dot{B}} \partial_{AB} - n_{A\dot{B}} \frac{1}{c}\frac{\partial}{\partial t} \tag{4}$$

where $\partial_{AB} = \partial_{BA}$ form a spatial triad [such that $\partial_{AB} \cdot \partial_{CD} = -\frac{1}{2}(\epsilon_{AC}\epsilon_{BD} + \epsilon_{BC}\epsilon_{AD})$]. In a similar manner, we can write

$$\psi_{AB\dot{C}} = -\sqrt{2}\, n^C{}_{\dot{C}} \psi_{BCA} - n_{B\dot{C}} \psi_A \tag{5}$$

where $\psi_{ABC} = \psi_{(AB)C}$ and the parentheses denote symmetrization on the indices enclosed. Substituting equations (4)–(5) into equation (1), one obtains

$$\sqrt{2}(\nabla_{AB}\psi_{CD}{}^A - \nabla_{CD}\psi_{AB}{}^A) - \frac{1}{c}\frac{\partial}{\partial t}\psi_{CDB} + \nabla_{CD}\psi_B = 0 \tag{6a}$$

$$\nabla_{AB}\psi^A - \frac{1}{c}\frac{\partial}{\partial t}\psi_{AB}{}^A = 0 \tag{6b}$$

Similarly, using equations (2) and (4)–(5), one finds that the supersymmetry transformations are given by

$$\psi_{ABC} \to \psi_{ABC} + \nabla_{AB}\alpha_C, \qquad \psi_A \to \psi_A + \frac{1}{c}\frac{\partial}{\partial t}\alpha_A \tag{7}$$

Equation (6a) is equivalent to

$$-\sqrt{2}\,\nabla^A{}_{(C}\psi_{D)AB} - \frac{1}{c}\frac{\partial}{\partial t}\psi_{CDB} + \nabla_{CD}\psi_B = 0 \tag{8}$$

and

$$\nabla_{A(C}\psi_{D)}{}^{AD} = 0 \tag{9}$$

It may be noticed that equation (9) can also be written as

$$\nabla_{AD}\psi_C{}^{AD} = \nabla_C{}^A\psi_{AD}{}^D \tag{10}$$

As a consequence of equations (6b) and (9), the spinor field

$$\phi_{ABC} \equiv \sqrt{2}\,\nabla^D{}_{(A}\psi_{B)DC} - \frac{1}{c}\frac{\partial}{\partial t}\psi_{ABC} + \nabla_{AB}\psi_C \tag{11}$$

is totally symmetric [cf. equation (8)]. Using the fact that, in flat space, $\nabla^D{}_A\nabla_{BD} = \frac{1}{2}\epsilon_{AB}\nabla^{CD}\nabla_{CD}$, one finds that ϕ_{ABC} is invariant under the gauge transformations (7). From equations (8) and (11) we have

$$\phi_{ABC} = 2\sqrt{2}\,\nabla^D{}_{(A}\psi_{B)DC} = -\frac{2}{c}\frac{\partial}{\partial t}\psi_{ABC} + 2\nabla_{AB}\psi_C \tag{12}$$

hence

$$\nabla^{AB}\phi_{ABC} = 0, \qquad \frac{1}{c}\frac{\partial}{\partial t}\phi_{ABC} = -\sqrt{2}\,\nabla^D{}_{(A}\phi_{BC)D} \tag{13}$$

which are the usual massless free field equations (see, e.g., Torres del Castillo, 1994b).

As shown in Torres del Castillo (1989b), equations (1) [or, equivalently, equations (6)] imply the existence of a conserved current. In fact, by an explicit computation, using equations (6) and (10) and their complex conjugates, one obtains the continuity equation

$$\frac{\partial \rho}{\partial t} = \nabla_{AB}J^{AB} \tag{14}$$

with

$$\begin{aligned} \rho &\equiv \hat{\psi}_{ABC}\psi^{BCA} + \hat{\psi}^A{}_B{}^B\psi_{AC}{}^C \\ J^{AB} &\equiv -c\{\sqrt{2}(\psi^{(A|CD|}\hat{\psi}_{CD}{}^{B)} + \hat{\psi}^{(A|CD|}\psi_{CD}{}^{B)} - \psi^{CD(A}\hat{\psi}_{CD}{}^{B)}) \\ &\quad + \psi^{C(AB)}\hat{\psi}_C + \hat{\psi}^{C(AB)}\psi_C - \psi^{(A}{}_C{}^{|C|}\hat{\psi}^{B)} - \hat{\psi}^{(A}{}_C{}^{|C|}\psi^{B)}\} \end{aligned} \tag{15}$$

where, for any spinor field $\psi_{AB\ldots L}$,

$$\hat{\psi}_{AB\ldots L} \equiv \overline{\psi^{AB\ldots L}} \tag{16}$$

(see Torres del Castillo, 1994a) and the indices between bars are excluded from the symmetrization. The continuity equation (14) implies, in the usual way, that

$$Q \equiv \int \rho d^3x = \int (\hat{\psi}_{ABC}\psi^{BCA} + \hat{\psi}^A{}_B{}^B\psi_{AC}{}^C)\, d^3x \tag{17}$$

is a constant. Even though ρ is not invariant under the gauge transformations (7), the integral (17) is gauge invariant. In fact, a straightforward computation making use of equation (10) shows that, under the transformation (7), ρ transforms as

$$\rho \to \rho + \nabla^{AB}\bigg(\hat{\psi}_{CAB}\alpha^C + \psi_{CAB}\hat{\alpha}^C + \hat{\psi}_{AC}{}^C\alpha_B + \psi_{AC}{}^C\hat{\alpha}_B + \frac{1}{2}(\hat{\alpha}_A\nabla_{BC}\alpha^C - \alpha_A\nabla_{BC}\hat{\alpha}^C + \hat{\alpha}^C\nabla_{CA}\alpha_B - \alpha^C\nabla_{CA}\hat{\alpha}_B)\bigg) \qquad (18)$$

i.e., the transformed ρ differs from the original by a divergence and therefore Q is invariant under the transformations (7).

From equations (15) and (16) it follows that $\overline{J_{AB}} = -J^{AB}$, which means that J_{AB} corresponds to a real vector field (Torres del Castillo, 1994a,b) and that ρ is real; however, ρ is not necessarily positive. In fact, using the decomposition $\psi_{ABC} = \psi_{(ABC)} + \frac{2}{3}\epsilon_{C(A}\psi_{B)D}{}^D$, one finds that

$$\rho = \hat{\psi}_{(ABC)}\psi^{(ABC)} - \frac{4}{3}\hat{\psi}_{AB}{}^B\psi^A{}_C{}^C \qquad (19)$$

which is the difference of two positive quantities [see equation (16)]. Nevertheless, the integral Q [equation (17)] is nonnegative [see equation (36) below].

We close this section by pointing out that from the second equation in (13) it follows that

$$\frac{1}{c}\frac{\partial}{\partial t}\nabla^{AB}\phi_{ABC} = -\frac{\sqrt{2}}{3}\nabla^D{}_C\nabla^{AB}\phi_{ABD} \qquad (20)$$

thus, if $\nabla^{AB}\phi_{ABC}$ vanishes at a particular time, then $\nabla^{AB}\phi_{ABC}$ will vanish at any subsequent time. Therefore, the first equation in (13) can be considered as an *initial condition* for the second equation in (13). [Analogously, the source-free Maxwell equations can be divided into evolution equations and initial conditions (see, e.g., Torres del Castillo and Acosta-Avalos, 1994.]

3. HAMILTONIAN STRUCTURE

Following Torres del Castillo and Acosta-Avalos (1994), in this section we shall give a Hamiltonian description for the gauge-independent equations (13). Assuming that the field ϕ_{ABC} satisfies periodic boundary conditions at the walls of a rectangular box of volume Ω, the components ϕ_{ABC} with respect to the basis induced by the Cartesian coordinates (Torres del Castillo, 1992) can be expressed as

$$\phi_{ABC}(\mathbf{x}, t) = \Omega^{-1/2}\sum_{\mathbf{k}}(a_{\mathbf{k}}(t)\lambda_A\lambda_B\lambda_C + b_{\mathbf{k}}(t)\hat{\lambda}_A\hat{\lambda}_B\hat{\lambda}_C)e^{i\mathbf{k}\cdot\mathbf{x}} \qquad (21)$$

where λ_A is such that the spinor equivalent of $\mathbf{k}$ is given by

$$k_{AB} = \sqrt{2}\,|\mathbf{k}|\,\lambda_{(A}\hat{\lambda}_{B)} \qquad (22)$$

[in other words, $\mathbf{k}/|\mathbf{k}|$ is the flagpole (or handle) of the flag (or ax) representing the spinor λ_A; see, e.g., Payne (1952) and Torres del Castillo (1990)]. Then, from equation (22) it follows that λ_A is normalized in the sense that

$$\lambda^A \hat{\lambda}_A = 1 \tag{23}$$

and

$$k_{AB}\lambda^A = \frac{|\mathbf{k}|}{\sqrt{2}} \lambda_A, \qquad k_{AB}\hat{\lambda}^B = -\frac{|\mathbf{k}|}{\sqrt{2}} \hat{\lambda}_A \tag{24}$$

The spinor λ_A can be taken in such a way that

$$(\lambda^A) = \begin{pmatrix} e^{-i\varphi/2}\cos(\theta/2) \\ e^{i\varphi/2}\sin(\theta/2) \end{pmatrix}, \qquad (\hat{\lambda}^A) = \begin{pmatrix} -e^{-i\varphi/2}\sin(\theta/2) \\ e^{i\varphi/2}\cos(\theta/2) \end{pmatrix}$$

where θ, φ are the polar and azimuth angles of $\mathbf{k}$, respectively.

Using the fact that, acting on $e^{i\mathbf{k}\cdot\mathbf{x}}$, the effect of ∂_{AB} is equivalent to multiplying by ik_{AB}, one finds that the expression (21) satisfies the first of equations (13) identically [in fact, this equation excludes the presence of terms proportional to $\lambda_{(A}\lambda_B\hat{\lambda}_{C)}$ and $\lambda_{(A}\hat{\lambda}_B\hat{\lambda}_{C)}$ in equation (21)] and from the second of equations (13), making use of equations (24), one obtains

$$\dot{a}_{\mathbf{k}} = i\omega a_{\mathbf{k}}, \qquad \dot{b}_{\mathbf{k}} = -i\omega b_{\mathbf{k}} \tag{25}$$

where a dot denotes partial differentiation with respect to the time and $\omega \equiv |\mathbf{k}|c$.

For each allowed vector $\mathbf{k}$ we introduce four real variables $q_{\mathbf{k}}$, $p_{\mathbf{k}}$, $\tilde{q}_{\mathbf{k}}$, and $\tilde{p}_{\mathbf{k}}$ satisfying the equations of motion

$$\dot{q}_{\mathbf{k}} = p_{\mathbf{k}}, \qquad \dot{p}_{\mathbf{k}} = -\omega^2 q_{\mathbf{k}}, \qquad \dot{\tilde{q}}_{\mathbf{k}} = -\tilde{p}_{\mathbf{k}}, \qquad \dot{\tilde{p}}_{\mathbf{k}} = \omega^2 \tilde{q}_{\mathbf{k}} \tag{26}$$

which follow from Hamilton's equations with the Hamiltonian

$$H = \frac{1}{2}\sum_{\mathbf{k}} [p_{\mathbf{k}}^2 + \omega^2 q_{\mathbf{k}}^2 - \tilde{p}_{\mathbf{k}}^2 - \omega^2 \tilde{q}_{\mathbf{k}}^2] \tag{27}$$

assuming that $q_{\mathbf{k}}$, $p_{\mathbf{k}}$ and $\tilde{q}_{\mathbf{k}}$, $\tilde{p}_{\mathbf{k}}$ are canonically conjugate variables. If we let

$$a_{\mathbf{k}} = (2\hbar\omega)^{-1/2}(p_{\mathbf{k}} + i\omega q_{\mathbf{k}}), \qquad b_{\mathbf{k}} = (2\hbar\omega)^{-1/2}(\tilde{p}_{\mathbf{k}} + i\omega\tilde{q}_{\mathbf{k}}) \tag{28}$$

then equations (25) are equivalent to equations (26); the only nonvanishing Poisson brackets among the expansion coefficients and their complex conjugates are given by

$$\{\overline{a_{\mathbf{k}}}, a_{\mathbf{k}'}\} = \{\overline{b_{\mathbf{k}}}, b_{\mathbf{k}'}\} = (i\hbar)^{-1}\delta_{\mathbf{k}\mathbf{k}'} \tag{29}$$

and the Hamiltonian (27) is

$$H = \sum_{\mathbf{k}} \hbar\omega(a_{\mathbf{k}}\overline{a_{\mathbf{k}}} - b_{\mathbf{k}}\overline{b_{\mathbf{k}}}) \tag{30}$$

On the other hand, from equations (21) and (23) one finds that the expansion coefficients $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ are given by

$$\begin{aligned} a_{\mathbf{k}} &= \Omega^{-1/2} \int \hat{\lambda}_A\hat{\lambda}_B\hat{\lambda}_C\hat{\phi}^{ABC}e^{-i\mathbf{k}\cdot\mathbf{x}}\, d^3x \\ b_{\mathbf{k}} &= \Omega^{-1/2} \int \lambda^A\lambda^B\lambda^C\phi_{ABC}e^{-i\mathbf{k}\cdot\mathbf{x}}\, d^3x \end{aligned} \tag{31}$$

Therefore, substituting equations (31) into equation (30), we find that the Hamiltonian is

$$H = -\int \hat{\phi}^{ABC}\sqrt{2}\, i\hbar c\nabla^R{}_A\phi_{BCR}\, d^3x \tag{32}$$

Equations (21) and (29) lead to the following Poisson brackets (at equal times)

$$\begin{aligned} \{\phi_{ABC}(\mathbf{x}, t), \phi_{DEF}(\mathbf{x}', t)\} &= 0 \\ \{\phi_{ABC}(\mathbf{x}, t), \hat{\phi}_{DEF}(\mathbf{x}', t)\} &= \frac{1}{i\hbar}\Omega^{-1}\sum_{\mathbf{k}} (\hat{\lambda}_A\hat{\lambda}_B\hat{\lambda}_C\lambda_D\lambda_E\lambda_F \\ &\quad - \lambda_A\lambda_B\lambda_C\hat{\lambda}_D\hat{\lambda}_E\hat{\lambda}_F)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \end{aligned} \tag{33}$$

The last equation can also be written in the form

$$\begin{aligned} &\{\phi_{ABC}(\mathbf{x}, t), \hat{\phi}_{DEF}(\mathbf{x}', t)\} \\ &\quad = -\frac{1}{i\hbar}\frac{\Omega^{-1}}{2}\sum_{\mathbf{k}} |\mathbf{k}|^{-2}\Big(k_{AD}k_{BE}\epsilon_{CF} + k_{AD}k_{CF}\epsilon_{BE} \\ &\qquad + k_{BE}k_{CF}\epsilon_{AD} + \frac{1}{2}|\mathbf{k}|^2\epsilon_{AD}\epsilon_{BE}\epsilon_{CF}\Big)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \end{aligned} \tag{34}$$

which shows that the left-hand side of equation (34) is symmetric in $\mathbf{x}$ and $\mathbf{x}'$.

The spinor potentials ψ_{BCD} and ψ_A corresponding to the field (21) can be chosen as

$$\begin{aligned} \psi_A &= 0 \\ \psi_{ABC} &= \frac{i}{2}\Omega^{-1/2}\sum_{\mathbf{k}} |\mathbf{k}|^{-1}(a_{\mathbf{k}}\lambda_A\lambda_B\lambda_C - b_{\mathbf{k}}\hat{\lambda}_A\hat{\lambda}_B\hat{\lambda}_C)e^{i\mathbf{k}\cdot\mathbf{x}} \end{aligned} \tag{35}$$

which satisfy equations (6) and (12). Substituting equation (35) into the definition (17), one obtains the expression

$$Q = \frac{1}{4} \sum_{\mathbf{k}} |\mathbf{k}|^{-2}(a_{\mathbf{k}}\overline{a_{\mathbf{k}}} + b_{\mathbf{k}}\overline{b_{\mathbf{k}}}) \tag{36}$$

which, by virtue of equations (25), is indeed time independent.

There exist many other real, gauge-invariant, conserved quantities that follow from equations (6) and (13). One of them is given by (Torres del Castillo, 1994b)

$$G \equiv \int \phi^{ABC}\hat{\phi}_{ABC}\, d^3x \tag{37}$$

which amounts to

$$G = \sum_{\mathbf{k}} (a_{\mathbf{k}}\overline{a_{\mathbf{k}}} + b_{\mathbf{k}}\overline{b_{\mathbf{k}}}) \tag{38}$$

[see equation (21)] and is clearly time independent [equations (25)]. From equations (29) and (38) it follows that

$$\{a_{\mathbf{k}}, G\} = -\frac{1}{i\hbar}\, a_{\mathbf{k}}, \qquad \{b_{\mathbf{k}}, G\} = -\frac{1}{i\hbar}\, b_{\mathbf{k}} \tag{39}$$

Therefore

$$\{\phi_{ABC}(\mathbf{x}, t), G\} = -\frac{1}{i\hbar}\, \phi_{ABC}(\mathbf{x}, t) \tag{40}$$

which means that G is the generator of the "duality rotations"

$$\phi_{ABC} \to e^{is/\hbar}\phi_{ABC} \tag{41}$$

that leave invariant the Hamiltonian (32) and the Poisson brackets (33) (Torres del Castillo and Acosta-Avalos, 1994).

The integral

$$C \equiv i \int \hat{\phi}^{ABC}\psi_{ABC}\, d^3x \tag{42}$$

is gauge invariant since, under the transformation (7),

$$\hat{\phi}^{ABC}\psi_{ABC} \to \hat{\phi}^{ABC}(\psi_{ABC} + \nabla_{AB}\alpha_C) = \hat{\phi}^{ABC}\psi_{ABC} + \nabla_{AB}(\hat{\phi}^{ABC}\alpha_C)$$

[equations (13)]. Therefore, we can evaluate the integral (42) making use of the specific choice (35), which gives

$$C = \frac{1}{2} \sum_{\mathbf{k}} |\mathbf{k}|^{-1}(a_{\mathbf{k}}\overline{a_{\mathbf{k}}} - b_{\mathbf{k}}\overline{b_{\mathbf{k}}}) \tag{43}$$

thus showing that C is real [even though the integrand in equation (42) is complex] and a constant of motion.

4. CONCLUDING REMARKS

As pointed out in Torres del Castillo (1989a), equations (1) are analogous to those of a self-dual electromagnetic field. In fact, the spinor potentials ψ_{ABC} and ψ_A are the analogs of the electromagnetic potentials $\mathbf{A}$ and φ, respectively [compare, e.g., equations (7) with the gauge transformations of the electromagnetic field], while ϕ_{ABC} is the analog of the gauge-invariant field $\mathbf{E} - i\mathbf{B}$ [see equation (11)]. Apart from the Hamiltonian structure given in Section 3, one can find an infinite number of alternative Hamiltonian structures by proposing linear relationships between $a_{\mathbf{k}}$, $b_{\mathbf{k}}$ and the canonical variables $q_{\mathbf{k}}$, $p_{\mathbf{k}}$, $\tilde{q}_{\mathbf{k}}$, $\tilde{p}_{\mathbf{k}}$ different from those given by equations (28) (see Torres del Castillo and Acosta-Avalos, 1994).

The use of the two-component spinor formalism is highly convenient, since it avoids the redundancies arising in the treatment of the Rarita–Schwinger field in terms of Dirac spinors. Another advantage of the two-component spinor formalism is that it allows one to deal with fields of any spin in a unified way; in fact, many of the expressions given in Section 3 are applicable to fields of any spin with minor changes.

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